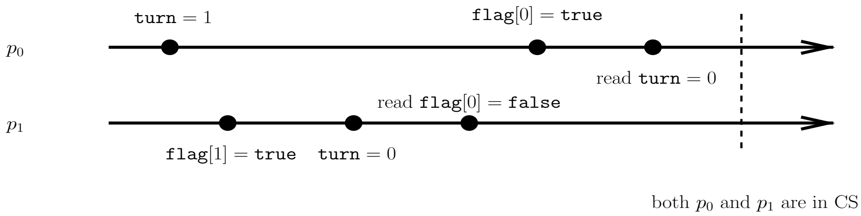
SE205: Solutions for Quiz 1

1 2-process Peterson's algorithm

Suppose that p_0 executes the first two lines of its algorithm in the reverse order:

1. `turn` $= 1$;
2. `flag`$[0] =$ `true`;

Then the following execution scenario is possible:



(Note that we do not care about the order in which the first two lines are executed by p_1.)
Here p_0 sets `turn` to 1, then p_1 sets `turn` to 0, `flag`$[0]$ to `true` (the order in which these two operations are performed does not matter) reads `false` in `flag`$[0]$ and proceeds to the critical section. Then p_0 reads 0 in `turn` and also proceeds to the critical section—a contradiction.

2 N-process Peterson's algorithm

Algorithm 1 N-process Peterson's algorithm

Shared variables:
  `level`$[0, \ldots, N-1] = \{-1\}$
  `waiting`$[0, \ldots, N-2] = \{-1\}$

Trying section: code for process p_i:
  for m from 0 to $N-2$ **do**
    `level`$[i] = m$;
    `waiting`$[m] = i$;
    `while(waiting`$[m] == i$ `&&` $(\exists\, k \neq i :$ `level`$[k] >= m)$`);`
Critical section:
  ...
Exit section:
  `level`$[i] = -1$;

Mutual exclusion. To prove that Algorithm 1 ensures the property of mutual exclusion, suppose, by contradiction, that it has an execution in which two processes are in their critical sections at some time t.

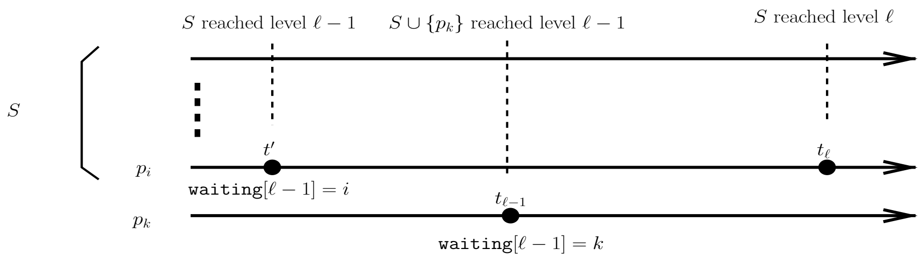
We say that a process p_i *reached level* ℓ ($\ell = 0, \ldots, N-1$) if it is in the critical section or `level`$[i]$ stores ℓ or a *higher* value. Thus, by our assumption, two processes reached level $N-1$ at the same time.

> Intuitively, a process that reached level ℓ is in the critical section or in the waiting phase ℓ or higher. By the algorithm, a process p_i executing its ℓ-th waiting phase should wait for every process that reached level ℓ to complete their critical sections, unless there is another process that wrote to *waiting*$[\ell]$ after p_i.

Suppose, inductively, that for some $\ell = N-1$ down to 1, a set S of $N-\ell+1$ processes reached level ℓ or higher at some time t_ℓ. (In the base case, $\ell = N-1$ and we have a set of 2 such processes.)

By the algorithm, before time t_ℓ, every process $p_i \in S$ sets `level`$[i]$ to $\ell-1$ and writes i in *waiting*$[\ell-1]$. Without loss of generality, assume that p_i is the last process in S to update *waiting*$[\ell-1]$ before t_ℓ, and let t' be the time when this happens. Hence, at time t', for every other process in $p_j \in S$, `level`$[j]$ stores $\ell-1$ or a higher value. Indeed, if at time t', for some process $p_j \in S$, `level`$[j]$ stores a value less than $\ell-1$, then to reach level ℓ by time t_ℓ, p_j must write j to `waiting`$[\ell-1]$ at some time between t' and t_ℓ, contradicting the assumption that p_i is the last process in S to write to `waiting`$[\ell-1]$ before t_ℓ.

Since $|S| = N-\ell+1$ and $\ell \leq N-1$, there is at least one process in S besides p_i. Thus, to reach level ℓ, between t' and t_ℓ, p_i must have read a value other than i in *waiting*$[\ell-1]$: otherwise, p_i would have to wait until all other processes in S complete their critical sections and set their `level` variables to -1. Thus, at some time $t_{\ell-1}$ between t' and t_ℓ, a process $p_k \notin S$ has written k in *waiting*$[\ell-1]$. Thus, at time $t_{\ell-1}$, at least $|S|+1 = N-\ell+2$ processes reached level $\ell-1$.



By induction, we derive that at some time t_0, at least $N+1$ process must have reached level 0, contradicting the fact that we have exactly N processes.

Starvation-freedom. Now we prove that Algorithm 1 ensures the property of starvation-freedom, i.e., assuming that no process fails in the trying, critical, or exit sections, every process in the trying section eventually enters its critical section. By the algorithm, the only possiblity for a process in the trying section not to enter its critical section is to *block* in line 8 at some level $\ell = 0, \ldots, N-2$. A process p_i blocks at level ℓ if, after setting `level`$[i]$ to ℓ and `waiting`$[\ell]$ to i, it keeps reading `waiting`$[\ell]$ and `level`$[0, \ldots, N-1]$ to always find `waiting`$[\ell]$ $== 1$ and `level`$[j] \geq \ell$ for some $j \neq i$. Since, prior to this, every process p_i writes i in `waiting`$[\ell]$, at most one process can be blocked at any given level.

Suppose, by contradiction that there exists a non-empty set B of blocked processes, and let p_I be the process that is blocked at the highest level ℓ. Let t be the time when p_i writes i to `waiting`$[\ell]$ for the last time. Thus, any process p_j that reaches level ℓ must have written j to `waiting`$[\ell]$ before t: otherwise, p_i would eventually read a value other than i and "unblock". Moreover, any such process that p_j must eventually complete level ℓ and proceed to the critical section: otherwise, it would block at a level higher than ℓ, violating our choice of p_i.

Thus, eventually, p_i would find out that no other process has reached level ℓ and proceed to level $\ell + 1$ or its critical section if $\ell = N - 2$—a contradiction.